tions $P_+(x)$ and $P_-(x)$ in the domain Ω, we can determine the topological picture of the distribution of the sets H_1, H_2, H_3 in Ω, and, in accordance with the Theorems 1.1, 1.4 and 1.5, we can classify the stability of the trivial solution of the system (1.2). Lemma 1.1 makes it possible to carry over this classification to the system (1.1).

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ASYMPTOTIC METHOD FOR MULTI-DIMENSIONAL SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS

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We consider almost linear symmetric hyperbolic systems with constant coefficients in the linear part and with nonlinear terms containing a small parameter. The asymptotic method used here for construction of approximate solutions is based on the work of Bogoliubov and Mitropol'skii [1], and has been applied to systems with a single independent spatial variable [2, 3]. Along with a slow time we introduce slow coordinates. For the approximate solution we obtain, not an infinite system as in [4], but a finite system of almost linear partial differential equations with constant coefficients, a system which is simpler than the original one. We present an algorithm for obtaining approximate solutions. We also show

that the approximate solution is close to the exact solution on an arbitrary finite interval.

1. We consider the system

$$u_t + \sum_{l=1}^{s} A^{(l)} u_l + Bu = \mu f(x,t) P(u) \tag{1.1}$$

$$u(x,t) = \{u^{(1)}, \ldots, u^{(n)}\}, \quad x = \{x_1, \ldots, x_s\}$$

$$u_l = \partial u / \partial x_l$$

$$f(x,t) = \{f^{(1)}, \ldots, f^{(n)}\}, \quad f \in C_{r+3}, \quad r > s/2 + 1$$

$$fP = \{f^{(1)}P^{(1)}, \ldots, f^{(n)} P^{(n)}\}$$

$$f(x,t) = \sum_{m=-\infty}^{\infty} f_m \exp\left[i\left(m_0 \nu t - \sum_{l=1}^{s} m_l \varkappa_l x_l\right)\right]$$

$$m = \{m_0, m_1, \ldots, m_s\}, \quad P = \{P^{(1)}, \ldots, P^{(n)}\}$$

Here $A^{(l)}$ and B are constant real square matrices of order n ; $A^{(l)}$ are symmetric matrices; μ is a small parameter, and $P(u)$ is a polynomial in u.

Along with the system (1.1) we consider the following degenerate linear system with constant coefficients

$$u_t + \sum_{l=1}^{s} A^{(l)} u_l + Bu = 0 \tag{1.2}$$

This system has solutions of the form:

$$u(x,t) = a\psi \exp[i(\omega t - kx)] + \text{c. c.}, \quad k = \{k_1, \ldots, k_s\} \tag{1.3}$$

(the abbreviation c. c. indicates the complex conjugate of the expression appearing before it). Here, a is an arbitrary complex number; ψ is a right zero vector of the matrix H ; ω and k are connected by means of the dispersion equation

$$D(\omega, k) = \det H = 0, \quad H = \omega I - \sum_{l=1}^{s} A^{(l)} k_l - iB \tag{1.4}$$

where I is the unit matrix.

Let there exist a nonempty set Ω of real frequencies ω_c mutually commensurable and also commensurable with ν ; let Ω also contain wave vectors $k_c = \{k_{c1}, \ldots, k_{cl}\}$ with real components, wherein like-named components $k_{1s}, \ldots, k_{cl}, \ldots$ are mutually commensurable and are also commensurable with $\varkappa_l$. The wave vector components satisfy the dispersion equation $D(\omega_c, k_c) = 0$. We assume also that

$$\partial D(\omega_c, k_c) / \partial \omega \neq 0 \tag{1.5}$$

Consequently, in neighborhoods of the points (ω_c, k_c) Eq. (1.4) defines ω as a single-valued function of k. Further, we assume that the values $d\omega / dk_l |_{k=k_{cl}}$ are real. To each set of values of (ω_c, k_c) there corresponds an eigen-wave of the system (1.1) of the form (1.3).

We pose the problem of constructing an approximate solution of the system (1.1), generated by an initial combination of waves of the form

$$\sum_{c=1}^{d} a_c^{(0)}\psi_c \exp(-ik_c x) + \text{c. c.}\,, \quad -\infty < x < \infty, \quad (\omega_c, k_c) \in \Omega \tag{1.6}$$

where $a_c^{(0)}$ are specified complex numbers.

We assume the following conditions to be satisfied.

1°. The set N_1 of pairs of vectors

$$(m^{(b)}, \ n^{(b)}), \quad m^{(b)} = \{m_0^{(b)}, m_1^{(b)}, \ldots, m_s^{(b)}\},$$
$$n^{(b)} = \{n_1^{(b)}, \ldots, n_d^{(b)}\}$$

with integral components, for which

$$(m_0^{(b)}\nu + n^{(b)}\omega, \ m_1^{(b)}\varkappa_1 + n^{(b)}k^{(1)}, \ldots, m_s^{(b)}\varkappa_s + n^{(b)}k^{(s)}) \in \Omega$$
$$b = d+1, \ldots, q, \quad n^{(b)}\omega = \sum_{c=1}^{d} n_c^{(b)}\omega_c, \quad n^{(b)}k^{(l)} = \sum_{c=1}^{d} n_c^{(b)}k_{cl}$$

is finite, i.e. the system has a finite number of combinative resonance waves $(\omega_c, k_c) \in \Omega$, $c = d+1, \ldots, q$.

2°. There exists a constant L_1 such that

$$\inf |D(m_0^{(b)}\nu + n^{(b)}\omega, \quad m_1^{(b)}\varkappa_1 + n^{(b)}k^{(1)}, \ldots,$$
$$m_s^{(b)}\varkappa_s + n^{(b)}k^{(s)})| = L_1 > 0, \quad \{m^{(b)}, n^{(b)}\} \equiv N_1$$

This requirement means that the number of combinative waves arbitrarily close to resonance waves is finite. (In practical calculations one should regard combinative waves for which $D(\omega, k) \sim \mu$ as resonance waves).

Conditions 1° and 2° are satisfied, in particular, speaking geometrically, the surface in E_{s+1} (the space of $\omega_0, k_1, \ldots, k_s$), defined by the dispersion equation, is bounded with respect to one of the coordinates.

We seek an approximate solution of the system (1.1) in the half-space $(-\infty < x < \infty, \ 0 \leqslant t < \infty)$, belonging to the class C_r, in the form

$$u^{(1)}(x,t) = \sum_{c=1}^{q} a_c(\chi, \tau)\psi_c \exp[i(\omega_c t - k_c x)] + \tag{1.7}$$
$$\text{c. c.} + \mu w(x, t, \chi, \tau), \quad -\infty < x < \infty, \quad 0 \leqslant t < \infty$$
$$\chi = \chi_0 + \mu x, \quad \tau = t_0 + \mu t$$

Here, χ_0 and τ_0 are constants; $a_c(\chi, \tau)$ are unknown scalar complex-valued functions of the slow variables χ and τ; $w(x, t, \chi, \tau)$ is an unknown real vector-valued function, periodic in x and t with the periods $\Lambda = \{\Lambda_1, \ldots, \Lambda_s\}$ and θ, respectively; Λ_l is the least common multiple of the numbers $2\pi / \varkappa_l$, $2\pi / k_{cl}$ $(c = 1, \ldots, d)$; θ is the least common multiple of the numbers $2\pi / \nu$, $2\pi / \omega_c$ $(c = 1, \ldots, d)$. Thus, in (1.7), besides the initial waves (ω_c, k_c), $c = 1, \ldots, d$, there are included the combinative resonance waves (ω_c, k_c), $c = d+1, \ldots, q$.

We obtain an equation for the function $w(x, t, \chi, \tau)$ by equating to zero the coefficient of μ upon substituting into the operator

$$N(u^{(1)}) = u_t^{(1)} + \sum_{l=1}^{s} A^{(l)}u_l^{(1)} + Bu^{(1)} - \mu f(x,t)\, P(u^{(1)})$$

in place of $u^{(1)}$ of the expression (1.7), taking into account the representation

$$P(u^{(1)}) = P(V + \mu w) = P(V) + \mu\varphi(x, t, a, w) \tag{1.8}$$

$$V = \sum_{c=1}^{q} a_c(\chi, \tau)\psi_c \exp[i(\omega_c t - k_c x)] + \text{c. c.}$$

Here $\varphi(x, t, a, w)$ is a periodic function of x and t (a trigonometric polynomial) and a polynomial in a, w and μ.

With this choice of w, the function $u^{(1)}$ will be an exact solution of the system

$$u_t^{(1)} + \sum_{l=1}^{s} A^{(l)}u_l^{(1)} + Bu^{(1)} = \mu f(x,t)P(u^{(1)}) + \mu^2 g^{(1)}(x,t,\mu) \tag{1.9}$$

$$g^{(1)}(x,t,\mu) = f(x,t)\varphi(x,t,a,w) - \frac{\partial w}{\partial\tau} - \sum_{l=1}^{s} A^{(l)}\frac{\partial w}{\partial\chi_l} \tag{1.10}$$

Here $f\varphi = \{f^{(1)}\varphi^{(1)}, \ldots, f^{(n)}\varphi^{(n)}\}$.

For determining w we obtain the linear system

$$w_t + \sum_{l=1}^{s} A^{(l)}w_l + Bw = h(x,t,\chi,\tau) \tag{1.11}$$

$$h(x,t,\chi,\tau) = f(x,t)P(V) - \left\{\sum_{c=1}^{q} \exp[i(\omega_c t - k_c x)] \times\right. \tag{1.12}$$

$$\left.\left[I\frac{\partial a_c}{\partial\tau} + \sum_{l=1}^{s} A^{(l)}\frac{\partial a_c}{\partial\chi_l}\right]\psi_c + \text{c. c.}\right\}$$

In the system (1.11) we regard the slow variables χ and τ as parameters, independent of x and t. It follows from (1.12) that $h(x, t, \chi, \tau)$ consists of two terms: a product of f by the trigonometric polynomial $P(V)$, in which the coefficients of the harmonics are polynomials in the complex amplitudes a_c, and a trigonometric resonace polynomial, the coefficients of the harmonics of which depend on $\partial a_c / \partial x$ and $\partial a_c / \partial\tau$. We write $h(x, t, \chi, \tau)$ in the form of a Fourier series and extract from it the resonance part

$$h(x,t,\chi,\tau) = \left\{\sum_{c=1}^{q} F_c(\chi,\tau)\exp[i(\omega_c t - k_c x)] + \right. \tag{1.13}$$

$$\left.\sum_a F_a(\chi,\tau)\exp[i(\omega_a t - k_a x)]\right\} + \text{к. с.}$$

Here

$$(\omega_c, k_c) \in \Omega, \quad \omega_a = m_0^{(b)}\nu + n^{(b)}\omega$$

$$k_a = m^{(b)}\varkappa + n^{(b)}k, \quad \{m^{(b)}, n^{(b)}\} \in N_1$$

$$F_{c,a}(x,\tau) = \frac{1}{\theta\cdot\Lambda_1 \cdots \Lambda_s}\int_0^\theta\int_0^\Lambda h(x,t,\chi,\tau) \times \tag{1.14}$$

$$\exp[-i(\omega_{c,a}t - k_{c,a}x)]\,dtdx$$

We seek a solution of the system (1.11) in the form

$$w(x,t,\chi,\tau)=\Big\{\sum_{c=1}^{q} w_c(\chi,\tau)\exp[i(\omega_c t-k_c x)]+ \qquad (1.15)$$
$$\sum_a w_a(\chi,\tau)\exp[i(\omega_a t-k_a x)]\Big\}+$$

Upon substituting Eqs. (1.13) and (1.15) into (1.11), we obtain linear algebraic systems for determining $w_{c,a}(\chi,\tau)$:

$$H_{c,a}w_{c,a}=F_{c,a} \qquad (1.16)$$

Since $D(\omega_a, k_a)\neq 0$, we can determine $w_a(\chi,\tau)$ uniquely. For the existnce of $w_c(\chi,\tau)$ it is necessary and sufficient that

$$(\zeta_c, F_c)=0, \quad c=1,\ldots,q \qquad (1.17)$$

Here ζ_c is an arbitrary right zero vector of the matrix H_c^*, conjugate to the matrix H_c. We restrict the discussion to the case in which the rank of the matrix H_c is equal to $n-1$. Then ζ_c is determined to within a factor and the solution of the system (1.16) has the form

$$w_c(\chi,\tau)=z_c(\chi,\tau)+c_c(\chi,\tau)\psi_c \qquad (1.18)$$

Here $z_c(\chi,\tau)$ is an arbitrary solution of (1.16) and $c_c(\chi,\tau)$ are arbitrary functions of χ and τ. In what follows we assume that these functions belong to the class C_{r+1}.

We obtain differential equations for determining the slow amplitudes $a_c(\chi,\tau)$ from the condition (1.17), upon taking into account (1.14) and (1.12), in the form

$$\frac{\partial a_c}{\partial\tau}+\sum_{l=1}^{s} v_{cl}\frac{\partial a_c}{\partial x_l}=f_c(a), \qquad v_{cl}=\frac{(\zeta_c, A^{(l)}\psi_c)}{(\zeta_c,\psi_c)} \qquad (1.19)$$

$$f_c(a)=\frac{1}{(\zeta_c,\psi_c)\,\theta\cdot\Lambda_1\ \ldots\Lambda_s}\int_0^\theta\int_0^\Lambda f(x\ \ t)\,P(V)\exp[-i(\omega_c t-k_c x)]\,dt\,dx$$

Here $f_c(a)$ is a polynomial in the amplitudes a_c of the same degree as P. The coefficients v_{cl} are components of the group velocities v_c of waves propagating with the amplitudes a_c. To show this, we differentiate the identity $(\zeta, H\psi)=0$ with respect to k_l ; in doing so, we regard ω as a function of $k_1,\ldots,k_s$ determined by the dispersion equation

$$(\partial\zeta/\partial k_l, H\psi)+(\zeta, I\partial\omega/\partial k_l\psi)-(\zeta, A^{(l)}\psi)+(\zeta, H\partial\psi/\partial k_l)=0$$

Since $H\psi=0$ and $(\zeta, H\partial\psi/\partial k_l)=(H^*\zeta, \partial\psi/\partial k_l)=0$, then

$$\partial\omega/\partial k_l=(\zeta, A^{(l)}\psi)/(\zeta,\psi)=-D_{kl}'(\omega,k)/D_\omega'(\omega,k)$$

Hence, for $k=k_c$ we obtain $v_{cl}=\partial\omega/\partial k_{l|k=k_c}$. By assumption, the values of $\partial\omega/\partial k_{l|k=k_c}$ are real. From (1.5) it follows that the v_{cl} are finite and the system (1.19) is hyperbolic.

The initial conditions follow from (1.6) and have the form

$$a_c(\chi,0)=a_c^{(0)}, \quad c=1,\ldots,d;\ a_c(\chi,0)=0, \quad c=d+1,\ldots,q \qquad (1.20)$$

The system (1.19), with the initial conditions (1.20), has a locally analytic solution. We assume that the problem (1.19), (1.20) has a soluti on in the class C_{r+2} in the domain $-\infty<\chi<\infty$, $0\leqslant\tau<\infty$. Then the function $w(x,t,\chi,\tau)$, defined by the series (1.15), belong to the class C_{r+1} in the (x, t)-halfspace.

The right-hand sides of the systems (1.16) are polynomials in a, $\partial a / \partial\tau$ and $\partial a / \partial\chi$, and are therefore $r+1$ times continuously differentiable. Consequently, with the function c_c (χ, τ) chosen from the class C_{r+1}, the Fourier coefficients $w_{c,a}(\chi, \tau)$ also belong to the class C_{r+1}.

We note that the ambiguity in determing $w(x, t, \chi, \tau)$, owing to the arbitrariness in $c_c(\chi, \tau)$, is of the order of μ^2 in an arbitrary bounded domain $G: \{x \in X,\ 0 \leqslant t \leqslant T$ (X is the bounded portion of the space $\{x_1, \ldots, x_s\}$). In fact

$$c_c(\chi, \tau) - c_c(\chi, 0) = \int_0^\tau \frac{\partial c_c}{\partial\tau}\,\partial\tau, \quad |c_c(\chi, \tau) - c_c(\chi, 0)| \leqslant \mu T L_2$$

Here L_2 is a constant which bounds $\partial c / \partial\tau$ in the domain G_μ: $\{\chi \in \mu X,\ 0 \leqslant \tau \leqslant \mu T\}$. Therefore, the ambiguity in determining $u^{(1)}$ is of the order of μ^2. It may be shown similarly that the ambiguity in determining $\partial^j u^{(1)} / \partial\chi^j$, $j = 1, \ldots, r$ is of the same order.

Convergence of the series (1.15) and the possibility of termwise differentiation of it $r+1$ times is determined from its nonresonance part. Differentiating systems (1.16) and solving them, we obtain

$$D_j w_a = H_a^{-1} D_j F_a, \quad j = 0, 1, \ldots, r+1$$

Here, the symbol D_j stands for the derivative of order j with respect to an arbitrary combination of the independent variables τ, $\chi_1, \ldots, \chi_s$. By virtue of the condition 2° there exists a constant L_3 such that

$$|D_j w_a| \leqslant L_3 |D_j F_a|$$

The coefficients F_a represent the product of the Fourier coefficients f_m of the function $f(x, t)$ by polynomials in the amplitudes and their derivatives up to the order $r+1$ with a finite set of coefficients. In the domain G_μ these polynomials are uniformly bounded by a constant. Therefore, the $D_j w_a$ decrease in the same way as the Fourier corfficients of the function $D_j f(x, t)$, i.e. they decrease faster than

$$1 / |m|^{r+3-j}, \qquad |m|^j = |m_0|^{j_0} \cdot |m_1|^{j_1} \ldots |m_s|^{j_s}$$
$$j_0 + j_1 + \ldots + j_s = j$$

Consequently, the series (1.15) and its derivatives to the order $r+1$ converge absolutely and uniformly in the domain G and $w(x, t, \chi, \tau) \in C_{r+1}$.

It follows from (1.7) that since $a_c \in C_{r+2}$ and $w \in C_{r+1}$, then $u^{(1)}(x,t) \in C_{r+1}$, and from Eq.(1.10) it follows that $g^{(1)}(x, t, \mu) \in C_r$.

Thus, the problem of obtaining an approximate solution is reduced to the integration of the almost linear system (1.19) and the solution of the algebraic systems (1.16). The system (1.19) is considerably simpler than the original system and its solution can be found, for example, by the method of successive approximations by integrating along the rays

$$\chi_l - v_{cl}\,\tau = \text{const}$$

$$a_c^{(m+1)}(\chi, \tau) = a_c^{(0)} + \int_0^\tau f_c\{a^{(m)}[\chi - v_c(\tau - \sigma), \sigma]\}\,d\sigma$$

2. We show now that the approximate solution $u^{(1)}(x, t)$ is close to the exact solution $u(x, t)$ in a bounded domain of the (x, t)-halfspace when the initial conditions coincide or are in close proximity.

Theorem. Let $u(x, t)$ be a solution of the system (1.1), $u^{(1)}(x, t)$ be a solution of the system (1.9), and $u(x, t) \in C_r$, $u^{(1)}(x, t) \in C_r$ for $-\infty < x < \infty, 0 \leqslant t < \infty$. Let

$$u(x, 0) = \varphi(x), \quad u^{(1)}(x, 0) = \psi(x), \quad \varphi, \psi \in C_r, \quad -\infty < x < \infty \qquad (2.1)$$

$$|\varphi(x) - \psi(x)| < \mu^2 K, \quad |D_k\varphi(x) - D_k\psi(x)| < \mu^2 K \qquad (2.2)$$

Here $k \leqslant r$; D_k is the derivative of order k with respect to an arbitrary conbination of the variables $x_1, \ldots, x_s$; K is a constant.

Then for any $T > 0$ and any bounded closed simply-connected domain X with a smooth boundary L, there exists a constant M such that

$$|u^{(1)}(x, t) - u(x, t)| < \mu^2 M, \qquad \forall\ x \in X, 0 \leqslant t \leqslant T$$
$$0 \leqslant \mu \leqslant \mu_0$$

To ease the writing we shall not indicate here the dependence of u, $u^{(1)}$, $g^{(1)}$, φ and ψ on μ. We assume that the initial functions $\varphi(x, \mu)$ and $\psi(x, \mu)$, and their derivatives $D_k\varphi(x, \mu)$ and $D_k\psi(x, \mu)$, $k \leqslant r$ are continuous functions of μ. By a theorem on the continuous dependence of the solution of an almost linear system on a parameter [5], the functions $u(x, t, \mu)$, $D_k u$, $u^{(1)}(x, t, \mu)$ and $D_k u^{(1)}$ are continuous functions of μ for $k \leqslant r$. The function $g^{(1)}(x, t, \mu)$, as a consequence of (1.10), also depends continuously on μ.

Proof. Differentiation of the systems (1.1) and (1.9) with respect to an arbitrary combination of the variables up to the order r inclusive leads to extended systems of the form

$$y_t + \sum_{l=1}^{s} R^{(l)} \frac{\partial y}{\partial x_l} + Qy = \mu f_1(x, t, y) \qquad (2.3)$$

$$y_t^{(1)} + \sum_{l=1}^{s} R^{(l)} \frac{\partial y^{(1)}}{\partial x_l} + Qy^{(1)} = \mu f_1(x, t, y^{(1)}) + \mu^2 g_1(x, t)$$

Here y and $y^{(1)}$ are vectors whose components are components of the vectors u and $u^{(1)}$ respectively, and their derivatives up to order r inclusive, with respect to an arbitrary combination of the variables $x_1, \ldots, x_s$; $R^{(b)}Q$ are cell-like square matrices formed from the matrices A^l amd B, respectively. The vector-valued function f_1 is the sum of products of periodic functions of x and t by polynomials in y. The components of the vector-valued function $g_1(x, t)$ are components of the vector-valued function $g^{(1)}(x, t)$ and its derivatives up to order r inclusive, with respect to an arbitrary combination of the variables $x_1, \ldots, x_s$.

From the relations (2.1) and (2.2), we have $|y(x, 0) - y^{(1)}(x, 0)| < \mu^2 K$. We now consider the systems (2.3) as identities in x and t, obtained by substituting into the equations (2.3) their solutions (these solutions are assumed to satisfy the initial conditions given by the relations (2.1)). We subtract the first of the relations (2.3) from the second and consider the resulting identity. By a lemma of Hadamard on finite increments, we have

$$f_1(x, t, y^{(1)}) - f_1(x, t, y) = \varepsilon(x, t, y^{(1)}, y)(y^{(1)} - y) = \varepsilon(x, t)\, v(x, t)$$
$$v = y^{(1)} - y$$

The identity so obtained now assumes the form

$$v_t + \sum_{l=1}^{s} R^{(l)} \frac{\partial v}{\partial x_l} + (Q - \mu\varepsilon) v = \mu^2 g_1(x, t) \tag{2.4}$$

The elements of the matric function $\varepsilon(x, t)$ are sums of products of derivatives of the periodic function $f(x, t)$ up to order r inclusive, by polynomials in $y^{(1)}$ and y, and they are, therefore, continuous functions in the (x, t)-halfspace. Since the function $\varepsilon(x, t)$ is assumed to be known, we can consider (2.4) as a linear system and use the results given by Courant in [5] for estimating the norm of the solution of a symmetric hyperbolic system in terms of the norm of the initial function $v(x, 0)$ and the norm of the right-hand side. We consider the conoid of dependence for the set of points of the space $(t, x_1, \ldots, x_s)$ for $0 \leqslant t \leqslant T$ $(x_1, \ldots, x_s) \in X$. Let $R(h)$ be the section of this conoid by the plane $t = h$. We then have

$$\| v(h) \|^2 = \int_{R(h)} v^2(x, h)\, dx \leqslant C_1 \| v(0) \|^2 + C_2 \mu^4 \int_0^h \| g_1(\tau) \| \, d\tau \tag{2.5}$$

By virtue of (2.2), we have $\| v(0) \|^2 < \mu^4 C_3$ therefore, from the relation (2.5) we have

$$\| v(h) \|^2 < \mu^4 C_4, \quad 0 \leqslant h \leqslant T$$

By Sobolev's imbedding theorem [6] it then follows that

$$| u^{(1)}(x, t) - u(x, t) | < \mu^2 M, \quad 0 \leqslant t \leqslant T, \quad x \in X$$

This completes the proof of the theorem.

The magnitude of the constant M depends on the measure of the set X (being proportional to it); it contains the factor $e^{\alpha T}$, where α depends on the coefficients of the system and characterizes the stability of the system; it also depends on the maximum of the function $g^{(1)}(x, t)$.

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